Dividing Polynomials

When you divide a polynomial f(x) by a divisor d(x), you get a quotient polynomial q(x) and a remainder r(x). We write this as: $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$

The degree of the remainder must be less than the degree of the divisor.

We will first use a method called **polynomial long division**.

Divide $2x^4 + 3x^3 + 5x - 1$ by $x^2 - 2x + 2$.

$$2x^2 + 7x + 10$$

$$x^2 - 2x + 2 \overline{)\,2x^4 + 3x^3 + 5x - 1}$$

$$2x^4 - 4x^3 + 4x^2$$

$$7x^3 - 4x^2 + 5x$$

$$7x^3 - 14x^2 + 14x$$

$$10x^2 - 9x - 1$$

$$10x^2 - 20x + 20$$

$$11x - 21$$

$$\boxed{2x^2 + 7x + 10 + \frac{11x - 21}{x^2 - 2x + 2}}$$

Let $f(x) = 3x^3 - 2x^2 + 2x - 5$

Use long division to divide f(x) by x - 2. What is the quotient? What is the remainder?

$$3x^2 + 4x + 10$$

$$x - 2 \overline{)\,3x^3 - 2x^2 + 2x - 5}$$

$$3x^3 - 6x^2$$

$$4x^2 + 2x$$

$$4x^2 - 8x$$

$$10x - 5$$

$$10x - 20$$

$$15$$

$$3x^2 + 4x + 10 + \frac{15}{x-2}$$

Use synthetic substitution to evaluate f(2). How is f(2) related to the remainder? What do you notice about the other constants in the last row of the synthetic substitution?

$f(x) = 3x^3 - 2x^2 + 2x - 5$

2	3	-2	2	-5
	↓	6	8	20
	3	4	10	15
	x^2	x	C	R

$$3x^2 + 4x + 10 + \frac{15}{x-2}$$

$$5x^2 - x + 3$$

$$x^2 + 3x\ \overline{)\,5x^4 + 14x^3 + 9x}$$

$$\underline{5x^4 + 15x^3}$$

$$-x^3 + 9x$$

$$\underline{-x^3 - 3x^2}$$

$$3x^2 + 9x$$

$$\underline{3x^2 + 9x}$$

$$0$$

In the actitvity you should have discovered that f(2) gives you the remainder when f(x) is divided by x - 2. This result is generalized in the remainder thereom.

REMAINDER THEOREM

If a polynomial f(x) is divided by $\boxed{x - k,}$ then the remainder is r = $\boxed{f(k).}$

$x-2$

$x-k$

$x-(-3)$

You may also have discovered in the activity that synthetic substitution gives the coefficients of the quotient. For this reason, synthetic substitution is sometimes called **synthetic division**. It can be used to divide a polynomial by an expression of the form x - k.

Divide $x^3 + 2x^2 - 6x - 9$ by $x - 2$ ... Using Syn. Div.

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -6 & -9 \\ & \downarrow & 2 & 8 & 4 \\ \hline & 1 & 4 & 2 & -5 \end{array}$$

$$\boxed{x^2+4x+2+\frac{-5}{x-2}}$$

Divide $x^3 + 2x^2 - 6x - 9$ by $x + 3$ ... Using Syn. Div.

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -6 & -9 \\ & & -3 & 3 & 9 \\ \hline & 1 & -1 & -3 & 0 \end{array}$$

$$\boxed{x^2-x-3}$$

Assignment:

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