Using Properties of Exponents (a^n)

Recall that the expression a^n, where 'n' is a positive integer, represents the product that you obtain when 'a' is used as a factor 'n' times. In the following activity you will investigate two properties of exponents.

Products and Quotients of Powers

- How many factors of 2 are there in the product $(2^3)(2^4)$? Use your answer to write the product as a single power of 2.

$(2\cdot2\cdot2)(2\cdot2\cdot2\cdot2) = 2^7$

- Write each product as a single power of 2 by counting the factors of 2. Use a calculator to check you answers.

$(2^2)(2^5)$ 2^7 $(2^1)(2^6)$ 2^7 $(2^3)(2^6)$ 2^9 $(2^4)(2^4)$ 2^8

- Complete this equation: $(2^m)(2^n) = 2^?$ 2^{m+n}

- Write each quotient as a single power of 2 by first writing the numerator and denominator in "expanded form" (for example, $2^3 = (2)(2)(2)$) and then canceling commom factors. Use a calculator to check your answers.

$\frac{2^3}{2^1}$ 2^2 $\frac{2^5}{2^2}$ 2^3 $\frac{2^7}{2^3}$ 2^4 $\frac{2^6}{2^2}$ 2^4

- Complete this equation: $\frac{2^m}{2^n} = 2^?$ 2^{m-n}

Properties of Exponents

Let 'a' and 'b' be real numbers and let 'm' and 'n' be integers.

Product of Powers Property $(a^m)(a^n) = a^{m+n}$

Power of a Power Property $(a^m)^n = a^{mn}$

Power of a Product Property $(ab)^m = a^m b^m$

Negative Exponent Property $a^{-m} = \frac{1}{a^m}$, $a \neq 0$

Zero Exponent Property $a^0 = 1$, $a \neq 0$

Quotient of Powers Property $\frac{a^m}{a^n} = a^{m-n}$

Power of a Quotient Property $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

The properties of exponents can be used to evaluate numerical expressions and to simplify algebraic expressions.
A simplified algebraic expression contains only positive exponents.

Evaluating Numerical Expressions

$(2^3)^4 = 2^{12} = 4096$ Power of Power

$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$ Power of Quo.

$(-5)^{-6}(-5)^4 = (-5)^{(-6+4)}$ Prod. of Power

$= (-5)^{(-2)}$ Neg. Ex. Prop

$= \frac{1}{(-5)^2} = \frac{1}{25}$

Simplifying Algebraic Expressions

$\left(\frac{r}{s^{-5}}\right)^2 = \frac{r^2}{s^{-10}}$ Power of Quo. & Power of Power

$= r^2 s^{10}$ Neg. Ex. Prop.

$(7b^{-3})^2b^5b$

$49b^{-6}b^4$

$49b^0$

$49 \cdot 1 = 49$

$\frac{(xy^2)^2}{x^3y^{-1}} = \frac{x^2y^4y}{x^3} = \frac{y^5}{x^1} = y^5x^{-1} = \boxed{\frac{y^5}{x}}$

$\frac{\cancel{x} \cdot \cancel{x} y^5}{\cancel{x} \cdot \cancel{x} \cdot x}$

Using Properties of Exponents in Real Life

The radius of the sun is about 109 times as great as Earth's radius. How many times as great as Earth's volume is the sun's volume.

Volume of a sphere = $\frac{4}{3}\pi r^3$

Let 'r' represent Earth's radius.

$$\frac{\text{Sun's volume}}{\text{Earth's volume}} = \frac{\cancel{\tfrac{4}{3}}\cancel{\pi}(109r)^3}{\cancel{\tfrac{4}{3}}\cancel{\pi}r^3} = \frac{109^3\cancel{r^3}}{\cancel{r^3}} = 1{,}295{,}029$$

A number is expressed in **Scientific Notation** if it is in the form "c x 10" where $1 \leq c < 10$ and 'n' is an integer. For instance, the width of a molecule of water is about 2.5×10^{-8} meter, or 0.000000025 meter. When working with numbers in scientific notation, the properties of exponents can help make calculations easier.

Using Scientific Notation in Real Life

The red blood cells, white blood cells, and platelets found in human blood are all generated from the same stem cells. In laboratory experiments, scientists have found that as few as 10 stem cells can grow into 1,200,000,000,000 platelets in just four weeks. The number of white blood cells generated was 1/40 the number of platelets. How many white blood cells were generated?

Assignment:

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16-50 even