Using Cramer's Rule

You can use determinants to solve a system of linear equations. The method, called **Cramer's rule**, uses the **coeffecient matrix** of the linear system.

Linear System

$ax + by = e$

$cx + dy = f$

Coefficient Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Cramer's Rule for a 2 x 2 system

Let A be the coefficient matrix of this linear system:

$ax + by = e$

$cx + dy = f$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

If det A $\neq$ 0, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Cramer's Rule for a 2 x 2 system

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Use Cramer's rule to solve this system:

$8x + 5y = 2$

$2x - 4y = -10$

$(-1, 2)$

$$x = \frac{\begin{vmatrix} 2 & 5 \\ -10 & -4 \end{vmatrix}}{\begin{vmatrix} 8 & 5 \\ 2 & -4 \end{vmatrix}} = \frac{-8-(-50)}{-32-(10)} = \frac{42}{-42} = -1$$

$$y = \frac{\begin{vmatrix} 8 & 2 \\ 2 & -10 \end{vmatrix}}{-42} = \frac{-80-(4)}{-42} = \frac{-84}{-42} = 2$$

Extra Example

$2x + y = 1$

$3x - 2y = -23$

$(-3, 7)$

$$x = \frac{\begin{vmatrix} 1 & 1 \\ -23 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}} = \frac{-2-(-23)}{-4-(3)} = \frac{21}{-7} = -3$$

$$y = \frac{\begin{vmatrix} 2 & 1 \\ 3 & -23 \end{vmatrix}}{-7} = \frac{(-46)-3}{-7} = \frac{-49}{-7} = 7$$

Cramer's Rule for a 3 x 3 system

Let A be the coefficient matrix of this linear system:

$ax + by + cz = j$

$dx + ey + fz = k$

$gx + hy + iz = l$

If det A $\neq$ 0, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A} \qquad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A} \qquad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$$

Assignment:

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~~46(set-up only~~)

Quiz 4.1-4.3 Tomorrow