

Solving Linear Systems Algebraically
(TWO Methods)

The Substitution Method

- Step 1: Solve one of the equations for one of its variables.
- Step 2: Substitute the expression from Step 1 into the other equation and solve for the other variable.
- Step 3: Substitute the value from Step 2 into the revised equation from Step 1 and solve.

Example $3x + 4y = -4$

$$x + 2y = 2$$

Step 1: $x + 2y = 2$
 $x = -2y + 2$

Step 2: $3x + 4y = -4$
 $3(-2y + 2) + 4y = -4$
 $-6y + 6 + 4y = -4$
 $-2y + 6 = -4$
 $-2y = -10$
 $y = 5$

$x = -2(5) + 2$
 $x = -8$
 $(-8, 5)$

$$0 = 0$$

Inf. Many

$$0 = 1$$

No Sol.

Solve the linear system using the substitution method:

$$3x - y = 13$$

$$2x + 2y = -10$$

Method 2: The Linear Combination Method

- Step 1: Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables.
- Step 2: Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.
- Step 3: Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

Example: $2x - 4y = 13$
 $4x - 5y = 8$

Solve the linear system using the linear combination method:

$$\begin{cases} 2x - 6y = 19 \\ -3x + 2y = 10 \end{cases}$$

$$\begin{aligned} 2x - 6y &= 19 \\ -9x + 6y &= 30 \end{aligned}$$

$$\hline -7x = 49$$

$$x = -7$$

$$2x - 6y = 19$$

$$2(-7) - 6y = 19$$

$$-14 - 6y = 19$$

$$-6y = 33$$

$$y = -\frac{33}{6} = -\frac{11}{2}$$

$$\left(-7, -\frac{11}{2}\right)$$

The Linear Combination Method: Multiplying Both Equations-

$$7x - 12y = -22$$

$$-5x + 8y = 14$$

Linear Systems with Many or No Solutions

Solve the linear system:

$$x - 2y = 3$$

$$2x - 4y = 7$$

$$6x - 10y = 12$$

$$-15x + 25y = -30$$

Assignment:

Pg. 152-155

12-20 evens,
24-34 evens,
38-48 evens,
52, 54

Hint for # 52: First use the "two point" method to find the equations of the two diagonals. Then solve the linear system algebraically.